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Massive Gravity on a Non-extremal Brane

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ABSTRACT

We consider a brane world scenario which arises as the near-horizon region of a non-extremal D5-brane. There is a quasi-localized massive graviton mode, as well as harmonic modes of higher mass which are bound to the brane to a lesser degree. Lorentz invariance is slightly broken, which may have observable effects due to the leakage of the metastable graviton states into the bulk. Unlike for a brane world arising from an extremal D5-brane, there is no mass gap. We also find that a brane world arising from a non-extremal M5/M5-brane intersection has the same graviton dynamics as that of a non-extremal D5-brane. This is evidence that a previously conjectured duality relation between the dual quantum field theories of each p-brane background may hold away from extremality.

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1 Introduction

Randall and Sundrum [1, 2] have shown that, with fine tuned brane tension, a flat 3-brane embedded in AdS_5 can have a single massless bound state. Four-dimensional gravity is recovered at low-energy scales. It has also been proposed that part or all of gravitational interactions are the result of massive gravitons. For example, in one model, gravitational interactions are due to the net effect of the massless graviton and ultra-light Kaluza-Klein state(s) [3, 4, 5, 6]. In another proposal, there is no normalizable massless graviton and four-dimensional gravity is reproduced at intermediate scales from a resonance-like behavior of the Kaluza-Klein wave functions [5, 6, 7, 8, 9].

It has been shown that an AdS_4 brane in AdS_5 does not have a normalizable massless graviton. Instead, there is a very light, but massive bound state graviton mode, which reproduces four-dimensional gravity [10, 11, 12, 13]. The bound state mass as a function of brane tension, as well as the modified Newtonian gravitational potential, were explored in [14, 15].

Five-dimensional domain walls which localize gravity may arise from a sphere reduction from ten or eleven dimensions, as the near-horizon of extremal p-branes [16]. If such a p-brane is perturbed away from extremality, what effects would this have on the corresponding brane world? In this paper, we consider a brane world scenario which arises from a sphere reduction of the near-horizon region of a non-extremal D5-brane. This is a new example in which there is a massive bound graviton state. There is also the novel phenomenon of harmonic graviton modes of higher mass, which are bound to the brane to a lesser degree.

It has already been proposed that our observable world resides within the horizon of a non-extremal brane, in order to account for the small but nonzero entropy density of our universe [17, 18]. Hawking radiation would cause the brane to evolve into an extremal state, which could be a possible mechanism for the resolution of various problems associated with brane world scenarios, such as the observed flatness and approximate Lorentz invariance of our world. While our particular model embeds the brane world outside of the D5-brane horizon, much of the motivations of [17, 18] remain the same.

This paper is organized as follows. In section 2, we discuss how a deviation from extremality breaks Lorentz invariance, which may have observable effects due to the leakage of the (almost) massless graviton mode into the bulk [19]. In section 3, we show that there is a massive graviton quasi-localized on the brane world, as well as harmonic modes of higher mass that are bound to the brane to a lesser degree. Sections 2 and 3 focus on the brane world arising from the near-horizon region of a non-extremal D5-brane wrapped on T^2. In section 4, we consider a non-extremal M5/M5-brane intersection wrapped on T^3 or $K3$. In section 5, we have concluding remarks.

2 Non-extremality and violation of Lorentz invariance

The metric for a non-extremal D5-brane is given by

$$ds^2 = H^{-1/4}(-f dt^2 + dx_i^2) + H^{3/4}(f^{-1}dr^2 + r^2 d\Omega_3^2), \tag{2.1}$$

where

$$H = 1 + \frac{R^2}{r^2}, \qquad f = 1 - \frac{e^2 R^2}{r^2}, \tag{2.2}$$

and $i = 1, .., 5$. e is the non-extremality parameter. For $e \ll 1$, we can neglect the "1" in H in the near-horizon limit. If we make the coordinate transformation

$$\frac{r}{R} = \sqrt{\mathrm{e}^{-k|z|} + e^2}, \tag{2.3}$$

then the metric (2.1) in the near-horizon region can be written as

$$ds^2 = (\mathrm{e}^{-k|z|} + e^2)^{1/4}[f(-dt^2 + dz^2) + dx_i^2 + R^2 d\Omega_3^2]. \tag{2.4}$$

In the extremal limit $e = 0$, the metric (2.4) is expressed in the conformally-flat frame. $z = 0$ corresponds to $r/R = \sqrt{1 + e^2}$, which is the location of the brane-world. $z \to \infty$ corresponds to $r \to eR$, which was the horizon of the D5-brane before the near-horizon limit was taken. We will consider x_4 and x_5 to be wrapped around a compact manifold, so that the inhabitants of this brane world would observe their universe to be $1 + 3$-dimensional. That is, the D5-brane is dimensionally reduced on $T^2 \times S^3$.

Note that the metric in (2.4) does not exhibit four-dimensional Lorentz invariance in the brane world directions. However, the four-dimensional geometry on the brane world is (approximately) invariant under Lorentz symmetry, since at the brane world position $z = 0$ the factor in front of the dt^2 becomes $f(z = 0) = 1/(1 + e^2)$, which can readily be absorbed into the time coordinate [19].

The Laplacian on the non-compact world-volume coordinates is

$$\Box_4 = f^{-1}\partial_t^2 - \partial_{x_1}^2 - \partial_{x_2}^2 - \partial_{x_3}^2 = f^{-1}E^2 - \vec{p}^{\,2}. \tag{2.5}$$

The non-extremality breaks the bulk Lorentz invariance in such a way that the momentum term can be neglected at large z relative to the energy term. Thus, unlike the extremal limit in which there is a mass gap [16], the mass spectrum is continuous from $m = 0$. Furthermore, the massless graviton is no longer bound to the brane forever. This mode is quasi-localized for nonzero momentum. Such leakage into the extra dimension implies that the four-dimensional Lorentz invariance is only approximate [19, 21].

Another Lorentz-violating effect is the modification of the dispersion relation to

$$E^2 = m^2 + c^2\vec{p}^2, \tag{2.6}$$

where c depends on the spread of the wave function in the extra dimension [19, 20, 22], and $c = 1$ in the extremal case. For a prototype model of a scalar field $\phi_m(z)$ considered in [19], our non-extremal geometry yields the result

$$c^2 = 1 - \frac{e^2 k}{(1+e^2)^{3/2}} \frac{\int dzz|\phi_m(z)|^2}{\int dz|\phi_m(z)|^2}, \tag{2.7}$$

in the approximation of a narrow wave function.

3 Localization of massive graviton

The equation of motion for a graviton fluctuation is

$$\partial_M \sqrt{-g} g^{MN} \partial_N \Phi = 0, \tag{3.1}$$

where the wave function Φ depends on the non-compact coordinates t, z, x_1, x_2, x_3. We take $\Phi = \phi(z)M(t, x_1, x_2, x_3)$, where $\Box_{(4)}M = m^2 M$ and $\Box_4$ is the Laplacian on t, x_1, x_2, x_3.

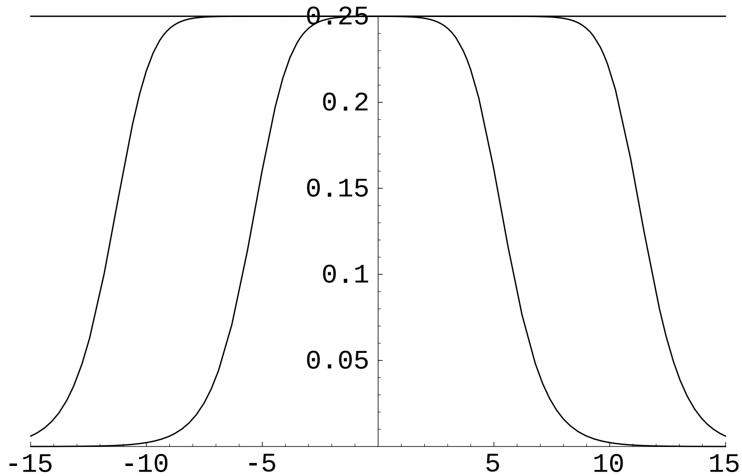

Figure 1: $V(z)$ for $m/k = 0$ and $e = 0, .005, .01$

For the background (2.4) the wave equation is

$$-\mathrm{e}^{k|z|}\partial_z(\mathrm{e}^{-k|z|} + e^2)\partial_z\phi = m^2\phi. \tag{3.2}$$

With the wave function transformation

$$\phi = (\mathrm{e}^{-k|z|} + e^2)^{-1/2}\psi, \tag{3.3}$$

the wave equation (3.2) becomes

$$-\partial_z^2\psi + \psi + V(z)\psi = 0, \tag{3.4}$$

where the effective potential

$$V(z) = \Big[\frac{k^2/2 - m^2}{1 + e^2\mathrm{e}^{k|z|}} - \frac{k^2}{4(1 + e^2\mathrm{e}^{k|z|})^2} - \frac{k}{1 + e^2}\delta(z)\Big], \tag{3.5}$$

which includes the mass term. For the massless mode, this is a Schrödinger-type wave equation, with the solution

$$\psi = N(e^2 + \mathrm{e}^{-k|z|})^{1/2}. \tag{3.6}$$

For the extremal case, this wave function is normalizable, with N being the normalization constant. The massless graviton is therefore bound to the extremal brane world.

The corresponding effective potential (3.5) is constant (for $e = 0$) with respect to z, as shown in Figure 1.

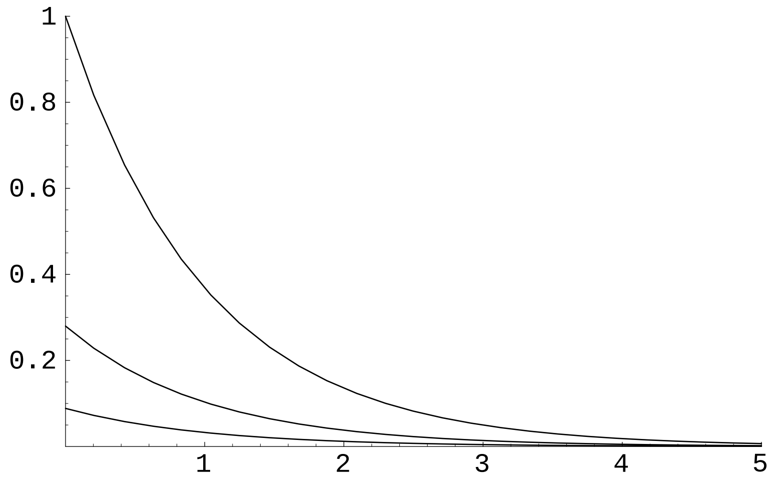

Figure 2: $|\psi(z)|^2$ for $m/k = 0$ and $e = 0, .005, .01$

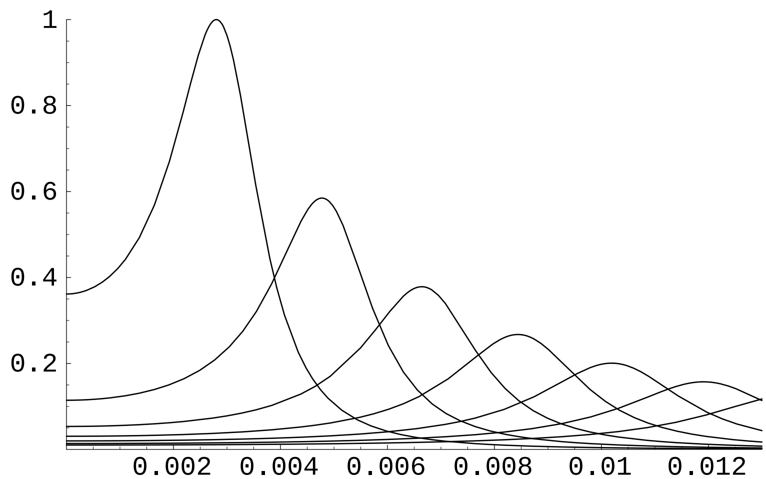

Figure 3: $|\psi(z = 0)|^2$ versus m/k for $e = .005, .01, .015, .02, .025, .03, .035$

For non-extremality, the effective potential decreases to zero as $|z| \to \infty$, as is depicted in Figure 1 for the cases $e = .005$ and $.01$. Away from extremality, ψ is not normalizable and the massless graviton propagates in the extra dimension z. This can be seen from Figure 2, in which the relative peak of $|\psi|^2$ at $z = 0$ is diminished away from extremality.

For the massive gravitons, the wave equation cannot be solved analytically and so we content ourselves with plotting numerical solutions. We numerically solve the wave equation outwards from $z = 0$ with the boundary conditions $\psi(z = 0) = 1$ (ψ is not yet normalized) and $\partial_z \psi(z = 0) = -k/(2 + 2e^2)$, the second of which is due

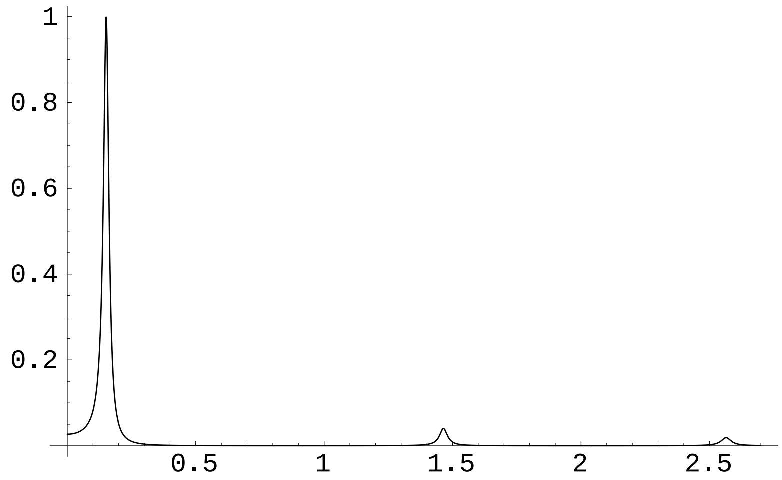

Figure 4: $|\psi(z=0)|^2$ versus m/k for $e = .5$

to the $\delta(z)$. We then numerically integrate in order to find the correct normalization factor. As can be seen in Figure 3, away from extremality there is a resonance in the wave function at nonzero mass, which represents a massive graviton mode localized on the brane world. The resonance mass increases as one moves away from extremality. Also, moving away from extremality has the effect of dissipating the wave function, which implies a decrease in the distance that the massive graviton propagates on the brane world before escaping into the extra dimension.

As the non-extremality parameter e increases, harmonic graviton states arise which are quasi-localized on the brane world, as is shown in Figure 4. However, for higher bound harmonics, the corresponding distance which the graviton mode travels on the brane world before escaping is diminished. The presence of these harmonic bound states is a novel feature in brane world scenarios.

4 D5-brane versus M5/M5-brane intersection

Supergravity domain-wall solutions with exponential scalar potentials can sometimes have the higher-dimensional origin as the near-horizon region of a p-brane configuration. It has been found that the near-horizon region of the extremal D5-brane reduced on $T^2 \times S^3$ yields the same exponential scalar potential as that of the extremal M5/M5-brane intersection reduced on $T^4 \times S^2$ or $K3 \times S^2$. This may hint that there is a duality relation between the quantum field theory living on the noncompact world volume of these two brane configurations [16].

We investigate this issue further by considering the non-extremal generalizations of the above p-brane configurations, from the point of view of the localization of gravity. Much of what has been written previously about the brane world arising from a non-extremal D5-brane also applies to the case of the non-extremal M5/M5-brane intersection. The metric for the latter is

$$\begin{aligned} ds^2 &= H_1^{-1/3} H_2^{-1/3}(-f dt^2 + dx_i^2) + H_1^{-1/3} H_2^{2/3} dy_j^2 + \\ &\quad H_1^{2/3} H_2^{-1/3} dw_j^2 + H_1^{2/3} H_2^{2/3}(f^{-1} dr^2 + r^2 d\Omega_2^2), \end{aligned} \tag{4.1}$$

where

$$H_i = 1 + \frac{R_i}{r}, \qquad f = 1 - \frac{gR}{r}, \tag{4.2}$$

and $i = 1, 2, 3$ and $j = 1, 2$. g is the non-extremality parameter. In the near-horizon limit, we can neglect the "1" in H_i and make the coordinate transformation

$$\frac{r}{R} = \mathrm{e}^{-k|z|} + g, \tag{4.3}$$

which is analogous to (2.3) for the D5-brane. The corresponding wave equation for a graviton fluctuation is

$$-\mathrm{e}^{k|z|}\partial_z(\mathrm{e}^{-k|z|} + g)\partial_z\phi = m^2\phi. \tag{4.4}$$

If we take $g \to e^2$, then the above wave equation is identical to that for the D5-brane given by (3.2). Thus, from the point of view of localization of gravity, a duality relation between the quantum field theory living on the world volume of the D5-brane wrapped on T^2 and that of the $M5/M5$-brane intersection wrapped on T^4 or $K3$ may hold away from extremality, provided that g is identified with e^2.

5 Conclusions

We have considered a brane world scenario which arises from the near-horizon region of a non-extremal D5-brane wrapped around T^2. Unlike for the extremal D5-brane, the graviton spectrum has no mass gap. There is a quasi-localized massive graviton mode, whose mass increases continuously from zero as one moves away from extremality. In addition, there are harmonic graviton modes which are bound to the brane to a lesser degree. In this scenario, Lorentz invariance is slightly broken, effects of which may be communicated from the bulk to the brane via the leakage of metastable graviton states. It would be interesting to see if the same graviton dynamics occur for a brane world arising from other p-brane origins, such as the near-horizon region of a D4-brane wrapped around S^1 or that of a D3-brane. However, in most cases the dynamical equations in non-extremal p-brane backgrounds are difficult to handle.

However, we do find the same graviton dynamics happening for a brane world arising from the near-horizon region of a non-extremal M5/M5-brane intersection wrapped around T^4 (or $K3$). In fact, this provides evidence that a conjectured duality relation between the dual quantum field theories living on the world volume of a D5-brane wrapped around T^2 and an M5/M5-brane intersection wrapped around T^4 may hold away from extremality.

This duality was initially conjectured because the dimensional reductions of the near-horizon regions of these two p-brane configurations down to five dimensions yield the same exponential scalar potential [16]. Such duality relations between various p-branes and p-brane intersections may exist for other cases. This is certainly worthy of further investigation, since there may be a more general class of examples for which a given gravity solution/QFT pair may be dual to one or more such pairs.

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